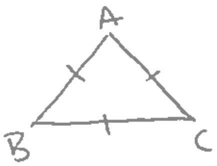
① Prove that the 3 angles of an equilateral triangle are all equal to each other. What adjustments would your proof require if you accepted Euclid's definition of an isosceles triangle?

Ⓐ



Given: $\triangle ABC$, where $AB = BC = AC$

Prove: $\angle A = \angle B = \angle C$

Proof: Since $\triangle ABC$ is equilateral, all of the sides are the same length, $AB = BC = AC$. Using Proposition 2.3.5, we can say that the base angles $\angle B + \angle C$ are equal to each other because the sides $AB = AC$. Since all of the sides are equal, we can use Proposition 2.3.5 again to say that if $BA = BC$, then $\angle A = \angle C$. Thus, by transitivity, $\angle A = \angle C$ and $\angle C = \angle B$, so $\angle A = \angle B$. Therefore, $\angle A = \angle B = \angle C$. □

B) Euclid's definition of isosceles triangle is, "Of trilateral figures, an equilateral triangle is that which has 3 sides equal, an isosceles triangle that which has two of its sides alone equal."

We will prove that the three angles of an equilateral triangle are all equal to each other. We are given an equilateral triangle. Consider $\triangle ABC$ with $AB = BC = AC$ by definition of equilateral triangle. Also consider $\triangle ACB$. We are given $AB = AC$. We know $\angle BAC = \angle BAC$ and $BC = BC$. By SAS, $\triangle ABC \cong \triangle ACB$. We know $\angle ABC = \angle ACB$. By SAS, $\triangle BAC \cong \triangle BCA$. We now know $\angle BAC = \angle BCA$. Therefore, $\angle ABC = \angle BAC = \angle BCA$ by transitivity. We have proven $\angle A = \angle B = \angle C$, as desired. ■